Probing Quantum Mechanics in the Macroregime using Macrorealist inequalities

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Some Fundamental Quantum Questions

1. To what extent it is possible to test QM in the macrolimit? Limits of observability of quantum effects in the macroregime?
2. How to reconcile our everyday experience of the macroscopic world with the weird behaviour of the microphysical world described by Quantum Mechanics (QM)? Under what conditions do classical laws emerge out of QM?
3. What do the nonclassical features of QM reveal about the nature of physical reality?

Some Significant Experimental Developments regarding the Probing of QM in the Macroregime

- Loophole-free violation of Bell inequality has been demonstrated for entangled electron spins involving separation of 1.3 kilometers.
- Satellite-based Quantum Teleportation is achieved over a distance of 1,400 km.
- Quantum Interference of C_{60} molecule (size $\sim$ 1nm) with mass $=$ 720 amu.
- Quantum Interference of C_{70} fullerene molecule with mass $=$ 840 amu and $C_{60}F_{48}$ Fluorofullerene molecule with mass $=$ 1632 amu.
- Quantum Interference of bigger biomolecules of size $\sim$ 2 nm and mass $\sim 2 \times 10^3$ amu.
- Quantum Interference of organic molecules PFNS10 and TPPF152 of size $\sim$ 6 nm with 430 atoms and masses up to $\sim 7 \times 10^3$ amu.
- Expt. tests of Macroscopic Quantum Coherence for SQUID systems involving superposition of micro to nano amperes current involving $\sim 10^{15}$ electrons flowing along clockwise and anticlockwise directions.

Candidate Systems for further Experimental Probing

- Large spin ($\sim 10 - 100$) molecules (e.g. $[Co(H_2O)_6]^{3+}$ complex, $[CoF_6]^{3-}$ complex, Mn_{12}-acetate) in magnetic field
- Nano-objects of mass $\sim 10^6 - 10^9$ amu trapped by laser fields (optical levitation) or in an ion-trap

The term 'macroscopic' here is taken to denote quantum effects involving large distance, or quantum effects involving systems with large values of the parameter such as mass or systems with high dimensionality.

Motivation for the present talk

Significance of studying QM predicted testable violation of macrorealism (MR):

Probing the validity of the notion of macrorealism (MR) provides a powerful means for testing QM in the macroregime.

- Violation of MR can be used as a tool for certifying quantumness or for revealing nonclassicality in a context that is usually thought to be entailing classical behaviour.
- Violation of MR can be invoked for ruling out a class of realist models.

Experimental setup for demonstrating this dual significance necessarily requires an unambiguous implementation of Negative Result Measurement.

QM violation of Macrorealism

Various necessary conditions of MR

- *Leggett-Garg inequality* (LGI)
- *Wigner's form of LGI* (WLGI)
- The *No-Signalling in Time* (NSIT) condition

These are formulated in terms of *time-separated* joint probabilities/correlation functions corresponding to successive measurement outcomes for *an individual system.*

Study of QM violation of MR in two different contexts

(a) For *large spin* system in uniform magnetic field subjected to *coarse-grained measurements.*

(b) For oscillating nano-objects of mass $\sim 10^6 - 10^9$ amu prepared in the *Schrödinger Coherent State* (the most "classical-like" of all quantum states) of a *linear harmonic oscillator.*

The notion of Macrorealism (MR)

- *Macrorealism* is the conjunction of the notions of *Realism* and *Noninvasive Measurability (NIM)*.
- *Realism*: At *any* instant, even when not measured, a system is in a *definite* one of the available states and all its observable properties have *definite values*.
- *Noninvasive Measurability (NIM)*: It is possible, at least in principle, to *determine which* of the states the system is in, *without affecting* the state or the system's subsequent evolution.
- The idea of NIM is sought to be experimentally implemented by using *Negative Result Measurement (NRM)*.

The idea of Negative Result Measurement (NRM)

- Consider a system evolving between two states, say, 1 (Q = +1) and 2 (Q = -1).
- We are interested to calculate the correlation function of measured values of Q at two different times (t_1 and t_2).
- The measuring apparatus be such that if $Q(t_1)$ is, say, $+1$, the probe is *triggered*, while if $Q(t_1) = -1$, it is not triggered $\Rightarrow$ no interaction between the system and the probe.

  The results of *those postselected runs* are used for which $Q(t_1) = -1$, followed by the measurement of Q at $t_2 \rightarrow$ These results used for determining the joint probabilities $P{-+}(t_1, t_2)$ and $P{--}(t_1, t_2)$.

The idea of Negative Result Measurement (NRM)

- One can then use a complementary setup so that for a value of $Q(t_1) = -1$ the probe is *triggered, while for* $Q(t_1) = +1$, it is not triggered.

  In this case, the results of *those postselected runs* are used for which $Q(t_1) = +1$, followed by the measurement of Q at $t_2 \rightarrow$ These results used for determining $P{+-}(t_1, t_2)$ and $P{++}(t_1, t_2)$.

- Using the four joint probability distributions determined in the above way, ensuring that the first measurement in each pair is noninvasive in the operational sense explained above, one can evaluate the correlation function of measured values of Q at t_1 and t_2.

Violation of Macrorealism(MR) using Negative Result Measurement (NRM)

- Empirical violation of the macrorealist condition observed using NRM not only certifies non-classicality, but also rules out a certain class of realist models defined within the framework of MR.
- Two experimental claims to date for loophole-free implementation of NRM in testing MR:

  G. Knee et al., *Nature Communications* 3, 606 (2012) → Spin-bearing phosphorus impurities in silicon sample.

  C. Robens et al. *Physical Review X* 5, 011003 (2015) → Quantum Walks in a lattice having cesium atoms.

  Criticisms persist about the possible loopholes in the claim of implementing NRM in the above mentioned experiments.

Various necessary conditions proposed for testing macrorealism

1. *Leggett-Garg Inequality (LGI)*: [A. J. Leggett and A. Garg, *PRL* **54**, 857 (1985)] $\rightarrow$ Derived as a testable algebraic consequence of the deterministic form of Macrorealism.

2. *Wigner's form of LGI(WLGI)*: [D. Saha, S. Mal, P. K. Panigrahi, D. Home, *PRA* **91**, 032117 (2015)] $\rightarrow$ Derived as a testable algebraic consequence of the probabilistic form of Macrorealism.

3. *No-Signalling in Time (NSIT)*: [J. Kofler and C. Brukner, *PRA* **87**, 052115 (2013)] $\rightarrow$ This condition is formulated as a statistical version of NIM to be satisfied by any macrorealist theory. Violation of NSIT implies violation of NIM at an individual macrorealist level.

Leggett-Garg Inequality (LGI)

- We consider temporal evolution for a two state system where the available states are, say, 1 and 2.
- Let Q(t) be on observable quantity such that, whenever measured, it is found to take a value+1 (-1) depending on whether the system is in 1 (2). Considering values of Q at three subsequent times $t_1 < t_2 < t_3$, it follows that

$$Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3) = +1 \text{ or } -3$$

whence one obtains for the 'grand' ensemble average

$$\langle Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3)\rangle_G \leq 1$$

Now, dividing the whole ensemble of runs into three subensembles, S_1, S_2, and S_3, consider measurement of Q on each subensemble of runs at the times (t_1, t_2) for S_1, (t_2, t_3) for S_2 and (t_3, t_1) for S_3 corresponding to the same initial state at $t = 0$.

One can then use the following deterministic consequence of the assumptions of realism and NIM:

For any set of runs corresponding to the *same initial state* at, say, $t = 0$, any individual $Q(t_i)$ has the same definite value, irrespective of the pair in which it occurs, i.e., the value of $Q(t_i)$ in any pair *does not* depend on whether *any prior* or *subsequent measurement* has been made on the system.

Leggett-Garg Inequality (LGI)

It then follows that

$$\langle Q(t_1)Q(t_2)\rangle_{S_1} + \langle Q(t_2)Q(t_3)\rangle_{S_2} - \langle Q(t_1)Q(t_3)\rangle_{S_3} \leq 1$$

where the grand ensemble average has been replaced by the respective subensemble averages.

The above inequality can be written as

$$C \equiv C_{12} + C_{23} - C_{13} \leq 1 \quad (1)$$

where the temporal correlation

$$C_{ij} = \langle Q(t_i)Q(t_j)\rangle$$

LHS of the inequality (1) is an experimentally measurable quantity. This is the Leggett-Garg inquality imposing macrorealist constraint on the temporal correlations pertaining to any two level system.

Wigner's form of Leggett-Garg inequality (WLGI)

Here again consider temporal evolution of a two state system where the available states are, say, 1 & 2 and consider measurement of Q at t_1, t_2 and t_3 ($t_1 < t_2 < t_3$).

Here the notion of realism implies the *existence of overall joint probabilities* $\rho(Q_1, Q_2, Q_3)$ pertaining to different combinations of definite values of outcomes for the relevant measurements.

The assumption of NIM implies that the probabilities of such outcomes would be *unaffected by measurements*. Hence, by appropriate marginalization, the observable probabilities can be obtained.

For example, the observable joint probability $P(Q_2+, Q_3-)$ of obtaining the outcomes +1 and -1 for the sequential measurements of Q at the instants t_2 and t_3, respectively, can be written as

$$P(Q_2+, Q_3-) = \sum_{Q_1=\pm 1} \rho(Q_1, +, -)$$

Similarly writing the other measurable marginal joint probabilities $P(Q_1-, Q_3-)$ and $P(Q_1+, Q_2+)$, we get

$$P(Q_1+, Q_2+) + P(Q_1-, Q_3-) - P(Q_2+, Q_3-) = \rho(+,+,+) + \rho(-,-,-) \quad (2)$$

Then invoking *non-negativity of the overall joint probabilities* occurring on the RHS of the above equation, the following form of WLGI is obtained in terms of three pairs of two-time joint probabilities.

$$P(Q_2+, Q_3-) - P(Q_1+, Q_2+) - P(Q_1-, Q_3-) \leq 0$$

Similarly, other forms of WLGI involving any number of pairs of two-time joint probabilities can be derived by using various combinations of the observable joint probabilities.

No-Signalling in Time (NSIT)

Statement: The measurement outcome statistics for any observable at any instant is *independent* of whether any prior measurement has been performed.

Consider a system whose time evolution occurs between two possible states. Probability of obtaining the outcome $+1$ for the measurement of a dichotomic observable Q at an instant, say, t_2 *without* any earlier measurement being performed, is denoted by $P(Q_2 = +1)$.

NSIT requires that $P(Q_2 = +1)$ should remain *unchanged* even when an earlier measurement is made at t_1

$$P(Q_2 = +1) = P(Q_1 = +1, Q_2 = +1) + P(Q_1 = -1, Q_2 = +1)$$

Experiments showing QM violations of MR

Experimental violations of LGI:

1. Superconducting qubit → Continuous Weak Measurements → Palacios-Laloy et al. [*Nat. Phys.* **6**, 442 (2010)]
2. Superconducting qubit → Weak/Semi-weak point Measurements → Groen et al. [*Phys. Rev. Lett.* **111**, 090506 (2013)]
3. Nitrogen-vacancy centre → Weak Measurements → George et al. [*Proc. Natl. Acad. Sci. USA* **110**, 3777 (2013)]
4. Nuclear magnetic resonance → Projective Measurements → Athalye et al. [*Phys. Rev. Lett.* **107**, 130402 (2011)], Souza et al. [*New J. Phys.* **13**, 053023 (2011)].
5. Nuclear magnetic resonance → Ideal Negative Measurements → Katiyar et al. [*Phys. Rev. A* **87**, 052102 (2013)].
6. Photons → Weak/Semi-weak point Measurements → Goggin et al. [*Proc. Natl. Acad. Sci. USA* **108**, 1256 (2011)], Dressel et al. [*Phys. Rev. Lett.* **106**, 040402 (2011)], Suzuki et al. [*New J. Phys.* **14**, 103022 (2012)].
7. Photons → Projective Measurements → Xu et al. [*Sci. Rep.* **1**, 101 (2011)].
8. Phosphorus impurities in Silicon → Ideal Negative Measurements → Knee et al. [*Nat. Commun.* **3**, 606 (2012)].
9. Three level NMR → Modified Ideal Negative Measurements → Katiyar et al. [*New J. Phys.* **19**, 023033 (2017)].

Experiments showing QM violations of MR

Experimental violation of NSIT:

1. Superconducting flux qubit → Knee et al. [*Nature Communications* **7**, 13253 (2016)] .

Experimental violation of WLGI:

There is no experiment till date demonstrating violation of WLGI.

QM violation of MR for large spin

The first work presented in this Talk - *QM violation of Macrorealism for large spin and its robustness against Coarse-grained measurements*

with Shiladitya Mal and Debarshi Das

Backdrop

Emergence of Classicality from QM

- Whether or to what extent classicality emerges from QM in *large dimensional system* has been much studied.
- For *large spin systems*, it was shown earlier that Bell-type local realist inequalities and LGI is violated under *ideal projective measurements*.
- On the other hand, for *large spin systems*, arguments based on large spin dynamics have been put forward to justify the emergence of classicality, if measurements are *coarse-grained* - J. Kofler and C. Brukner, PRL 99, 180403 (2007); PRL 101, 090403 (2008).

QM violation of MR for large spin

Motivation

No study yet probing emergence of classicality for higher dimensional quantum systems by modelling coarse-graining of measurements in a general way taking into account the *fuzziness* in *measuring* each eigenvalue and in *discriminating* between different eigenvalues.

The key result obtained

Our study reveals that by employing QM violation of MR as a tool classicality does *not* emerge in large limit of spin, whatever be the *unsharpness* and degree of *coarse-graining of the measurements*. For this purpose, employing the different necessary conditions of MR (LGI, WLGI and NSIT), their relative efficacy in demonstrating non-classicality is assessed – NSIT is found to be most effective in this specific context.

Specifying the Hamiltonian, initial condition and measurement times

- Consider a QM spin j system in a uniform magnetic filed of magnitude B_0 along the x direction. The relevant Hamiltonian is ($\hbar = 1$)

$$H = \Omega J_x$$

where $\Omega \rightarrow$ angular precession frequency ($\propto B_0$), $J_x \rightarrow$ x component of spin angular momentum.

- We initialize the system so that at t=0, the system is in the state $|-j;j\rangle$; where $|m;j\rangle$ denotes the eigen state of J_z operator with eigenvalue m.
- Consider measurements of Q at times t_1, t_2 & t_3 ($t_1 < t_2 < t_3$) & set the measurement times as $\Omega t_1 = \pi$ and $\Omega(t_2 - t_1) = \Omega(t_3 - t_2) = \frac{\pi}{2}$
- Now, consider the following form of 3-term LGI:

$$K_{LGI} = C_{12} + C_{23} - C_{13} \leq 1$$

The following form of 3-term WLGI:

$$K_{WLGI} = P(Q_2+, Q_3+) - P(Q_1-, Q_2+) - P(Q_1+, Q_3+) \leq 0$$

and the following form of NSIT:

$$K_{NSIT} = P(Q_3 = -1) - [P(Q_2 = +1, Q_3 = -1) + P(Q_2 = -1, Q_3 = -1)] = 0$$

Modelling coarse grained measurement in an arbitrary spin system

Considering measurements of spin-z component (J_z) observable in a spin-j system, the outcomes of J_z measurements are denoted by m, m takes the values $-j$, $-j+1$, $-j+2$, ..., $+j$. For modelling coarse grained measurement through appropriate dichotomization, different number of measurement outcomes are clubbed together into two groups, the grouping scheme being characterized by a particular value of x.

Let Q be such an observable that

$Q = -1$ for $m = -j, ..., -j + x$ (No. of outcomes in this group $= x + 1$)

$Q = +1$ for $m = -j + x + 1, ..., +j$ (No. of outcomes in this group $= 2j - x$)

Equal or almost equal number of outcomes in the two groups denotes the grouping scheme corresponding to the *unbiased* coarse-graining of the measurement outcomes.

Modelling in a general way coarse grained measurement for an arbitrary spin system

• (a) Modelling fuzziness of measurement for each eigenvalue through sharpness parameter using POVM.

(b) Clubbing of the measurement outcomes into two groups and by varying the number of outcomes in each group.

• In this way by invoking unsharp measurement and clubbing the different measurement outcomes together one can capture in a *general way* what is entailed by the *coarse-grained measurement*.

Modelling fuzziness of a measurement through sharpness parameter in a two-level system

Consider measurements involving a two-state system with states $|A\rangle$ and $|B\rangle$. Note that for the ideal measurement of the dichotomic observable $Q = |A\rangle\langle A| - |B\rangle\langle B|$, the respective probabilities of the outcomes ± 1 and the way a measurement affects the observed state are determined by the projection operators $P_{\pm}$ onto the state $|A\rangle$ ($|B\rangle$).

In order to capture the effect of fuzziness or imprecision involved in a measurement, in the formalism of unsharp measurement, a parameter (λ) known as the sharpness parameter is introduced to characterize non-idealness of a measurement by defining what is referred to as the effect operator given by

$$F_{\pm} = \lambda P_{\pm} + (1 - \lambda)\mathbb{I}/2$$

where $\mathbb{I} = |A\rangle\langle A| + |B\rangle\langle B|$. It is clear that $(1 - \lambda)$ amount of white noise is present in the measurement, where $0 < \lambda \leq 1$ and $F_{\pm}$ are mutually commuting Hermitian operators with non-negative eigenvalues; $F_+ + F_- = \mathbb{I}$.

For an unsharp measurement pertaining to an initial state ρ_0, the probability of an outcome, say, $+1$ is given by $tr(\rho_0 F_+)$ for which the post-measurement state is given by $(\sqrt{F_+}\rho_0\sqrt{F_+})/tr(\rho_0 F_+)$.

Modelling unsharp measurement in an arbitrary spin system

Consider measurements of spin-z component (J_z) observable in a spin-j system. In the formalism of POVM, to characterize the non-idealness of a measurement, the effect operators are given by

$$F_m = \lambda P_m + (1 - \lambda)\frac{\mathbb{I}}{d}$$

where λ is the sharpness parameter ($0 < \lambda \leq 1$);

P_m is the projector $|m; j\rangle\langle m; j|$, where $|m; j\rangle$ is the eigenvector of J_z operator with eigenvalue m;

$\mathbb{I}$ is the identity operator,

d is the dimension of the system (for spin j system, $d = 2j + 1$)

Summary of the Key Results

Considering unsharp measurement ($0 < \lambda \leq 1$)

By varying the number of measurement outcomes in the two groups used in the general model of coarse-graining, the results for even the most extreme coarse-graining of outcomes show that

• LGI → not violated for any spin-*j* system.

• WLGI → QM violation persists up to a *certain* degree of fuzziness of the measurement for any spin-*j* system. Example: for spin $j = 30$ system, QM violation of WLGI persists in the range $(0.75, 1]$ of λ.

• NSIT → QM violation persists for *any* degree of fuzziness of the measurement for *any* spin-*j* system.

Essence of the Results

These results signify that, in the limit of arbitrarily large spin system, even using a general model of coarse-graining of the measurement outcomes, classicality does *not* emerge for *any* degree of fuzziness of the measurement. This is best illustrated through the QM violation of NSIT.

S. Mal, D. Das and D. Home; *Physical Review A* **94**, 062117.

Feasibility of experimental study of this feature being explored with appropriate large spin molecules.

Nonclassicality of the Harmonic-Oscillator Coherent State persisting up to the Macroscopic Domain

A proposed empirically testable setup

Dipankar Home with Sougato Bose and Shiladitya Mal

Motivation

- Using violation of macrorealism as a tool, to show that the most "classical-like" of all quantum states, viz. the Schrödinger coherent state of a harmonic oscillator can exhibit non-classical behaviour that persists up to *large mass values*.
- This would enable testing whether recently engineered *macroscopic quantum oscillators* like trapped *oscillating nano-crystals* are bonafide nonclassical objects, *without* using nonlinearity or coupling with any ancillary quantum system, and *without* requiring initial preparation of Schrödinger cat type state.

The key result obtained

It is found that for any given mass and oscillator frequency, a significant amount of QM violation of macrorealism can be obtained by suitably choosing the initial peak momentum of the coherent state wave packet - empirical implementation of this scheme feasible using *mirror-based levitation procedure* for *trapped, cooled nano-crystals* of masses 10^6 amu and above.

The Proposed Setup

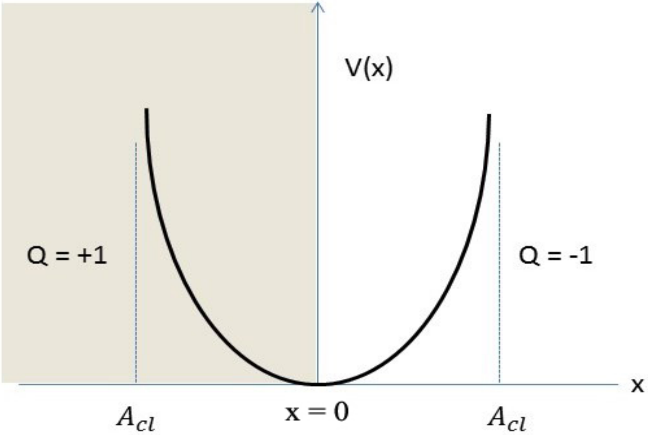
Other key features

(i) Implementation of noninvasive measurability (*NIM*) through negative result measurement (*NRM*) enabling testing of the everyday notion of MR in a loophole-free way for a *macroscopic system* having a *classical analogue*.

(ii) The example involves *continuous variables.* For probing MR, *discretization* is invoked by considering spatial measurement of the following type:

Coarse-grained measurement determining *which one of the spatial halves* of the region, the oscillating particle is *in* at any given instant.

THE SETUP



SCHEMATIC DESCRIPTION OF OUR WORK

Linear Harmonic Oscillator

- Initial wavepacket is

$$\psi(x,0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_0}} \exp\left(-\frac{x^2}{4\sigma_0^2} + \frac{ip_0 x}{\hbar}\right) \tag{3}$$

- If the particle is found in the region between $x \to -\infty$ and $x = 0$, then the measurement outcome is denoted by $+1$. If the particle is found in the region between $x = 0$ and $x \to \infty$, then the outcome is denoted by -1.
- The above mentioned condition is satisfied by defining the following measurement operator

$$\hat{O} = \int_{-\infty}^{0} |x\rangle\langle x| dx - \int_{0}^{\infty} |x\rangle\langle x| dx \tag{4}$$

PROPERTIES OF THE OBSERVABLE $\widehat{O}$

- The observable $\widehat{O}$ has two eigenstates having eigenvalues $+1$ and -1 respectively. For the eigenvalue $+1$, we have the corresponding eigenstate defined by

$$\hat{O}\int_{-\infty}^{0}\langle x|\psi\rangle|x\rangle dx = +1\int_{-\infty}^{0}\langle x| \tag{5}$$

- For the eigenvalue -1 we have the corresponding eigenstate defined by

$$\hat{O}\int_{0}^{\infty}\langle x|\psi\rangle|x\rangle dx = -1\int_{0}^{\infty}\langle x|\psi\rangle|x\rangle dx \tag{6}$$

MEASUREMENT RESULTS AT TIME t

- Probability at time t of finding the particle in the region between $x \to -\infty$ and $x = 0$ is given by

$$P_+(t) = \int_{-\infty}^{0} |\psi(x,t)|^2 dx = \frac{1}{2}\left(1 - Erf(\frac{\langle x(t)\rangle}{\sqrt{2}|\sigma_t|})\right) \quad (7)$$

- Probability at time t of finding the particle in the region between $x = 0$ and $x \to \infty$ is given by

$$P_-(t) = \int_{0}^{\infty} |\psi(x,t)|^2 dx = \frac{1}{2}\left(1 + Erf(\frac{\langle x(t)\rangle}{\sqrt{2}|\sigma_t|})\right) \quad (8)$$

ERROR FUNCTION

- Error function is defined as

$$Erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t \exp(-z^2) dz \tag{9}$$

- Few properties of error function are

$$Erf(\infty) = 1 \tag{10}$$

$$Erf(-t) = -Erf(t) \tag{11}$$

POST-MEASUREMENT STATE AT TIME t

- When the particle is found at the instant t in the region between $x \to -\infty$ and $x = 0$, the *post-measurement state* is given by

$$|\psi_{+}^{PM}(t)\rangle = \int_{-\infty}^{0} \psi(x', t)|x'\rangle dx' \tag{12}$$

- When the particle is found at the instant t in the region between $x = 0$ and $x \to \infty$, the *post-measurement state* is given by

$$|\psi_{-}^{PM}(t)\rangle = \int_{0}^{\infty} \psi(x', t)|x'\rangle dx' \tag{13}$$

FURTHER EVOLUTION OF THE STATE AFTER 1st MEASUREMENT

- If $+1$ result is obtained at, say, $t = t_1$, then the post-measurement state under the harmonic oscillator potential evolves into the following state at the instant $t = t_2$

$$|\psi_{+}^{PM}(t_2)\rangle = \int_{-\infty}^{\infty} K(x', t_1; x, t_2)\psi(x', t_1)_{+}^{PM}|x'\rangle dx' \quad (14)$$

- If -1 result is obtained at, say, $t = t_1$, then the post-measurement state under the harmonic oscillator potential evolves into the following state at the instant $t = t_2$

$$|\psi_{-}^{PM}(t_2)\rangle = \int_{-\infty}^{\infty} K(x', t_1; x, t_2)\psi(x', t_1)_{-}^{PM}|x'\rangle dx' \quad (15)$$

JOINT PROBABILITIES AFTER THE 2nd MEASUREMENT

- Conditional Probability of finding the particle in the region between $x \to -\infty$ and $x = 0$ at the instant t_2 when $\pm$ result for the measurement of the localization operator $\widehat{0}$ has been obtained at the instant t_1 is given by

$$P_{\pm/+}(t_1, t_2) = \int_{-\infty}^{0} |\psi(x, t_2)_{\pm}^{PM}|^2 dx \tag{16}$$

- Similarly, the Conditional Probability of finding the particle in the region between $x = 0$ and $x \to \infty$ at the instant t_2 when $\pm$ result for the measurement of the localization operator $\widehat{0}$ has been obtained at the instant t_1 is given by

$$P_{\pm/-}(t_1, t_2) = \int_{0}^{\infty} |\psi(x, t_2)_{\pm}^{PM}|^2 dx \tag{17}$$

TEMPORAL CORRELATION FUNCTIONS

- The *temporal correlation function*, say, C_{12} occurring in the Leggett-Garg inequality is given by

$$C_{12} = P_{++}(t_1, t_2) - P_{+-}(t_1, t_2) + P_{--}(t_1, t_2) - P_{-+}(t_1, t_2) \quad (18)$$

- where $P_{++}(t_1, t_2)$ is the *joint probability* of finding the measurement outcomes $+1$, $+1$ at the respective times t_1 and t_2; similarly, $P_{+-}(t_1, t_2)$, $P_{--}(t_1, t_2)$, and $P_{-+}(t_1, t_2)$ denote the corresponding *joint probabilities*. Thus, by evaluating these *joint probabilities*, one can calculate the quantity C_{12}.

  In a similar way, the other temporal correlation functions C_{23}, C_{34}, C_{14} occurring in the *4-term LGI* can also be calculated, thereby checking the validity of the *4-term LGI*

$$|C_{12} + C_{23} + C_{34} - C_{14}| \leq 2$$

SCHRÖDINGER COHERENT STATE

Taking $\sigma_0 = \sqrt{\frac{\hbar}{2m\omega}}$ in $\psi(x, 0)$ corresponds to Schrödinger Coherent State.

- Probability density of this time-evolved state is given by

$$|\psi(x,t)|^2 = \sqrt{\frac{m\omega}{\hbar\pi}} \exp\left(-m\omega \frac{(x - \frac{p_0}{m\omega}\sin\omega t)^2}{\hbar}\right) \qquad (19)$$

- The probability density of this wave packet oscillates *without spreading* or *changing shape* with its *peak* following classical motion and $\Delta x \Delta p = \hbar/2$. Hence coherent state is regarded as the "best possible" *quasi-classical* quantum description of the motion of a linear harmonic oscillator.

SALIENT FEATURES OF CALCULATIONAL RESULTS

In our setup, the key parameters are p_0, ω where p_0 is the initial peak momentum (expectation value of momentum corresponding to the initial wave packet) and ω is the angular frequency of the corresponding classical oscillation. Suitably choosing p_0, ω and by appropriate tuning of $t, \Delta t$, the QM violation of LGI for a given mass (m) may be shown.

In the calculational results we present, using the 4-term LGI, p_0 and ω are throughout chosen such that the corresponding classical amplitude of oscillation ($A_{Cl} = p_0/m\omega$) ranges from 10^{-4} m to 10^{-10} m, $\Delta t = 2.4 \times 10^{-6}$ s and $t_1 = 1.5 \times 10^{-6}$ s where $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \Delta t$, with the time period (T) of oscillation $T = 3.14 \times 10^{-6}$ s corresponding to $\omega = 2 \times 10^6$ Hz.

QUANTUM VIOLATION OF LGI FOR LARGER MASSES

Tuning p_0 appropriately one can find violation of LGI at large masses.
$\omega = 2 \times 10^6$ Hz. $\sigma_0 = \sqrt{\frac{\hbar}{2m\omega}}$; $A_{Cl} = \frac{p_0}{m\omega}$

MASS (amu)	$\sigma_0(m)$	$p_0(Kgm/s)$	$A_{Cl}(m)$	C
10	3.9×10^{-8}	10^{-24}	10^{-4}	2.62
10^3	3.9×10^{-9}	10^{-23}	10^{-5}	2.58
10^6	1.2×10^{-10}	10^{-21}	10^{-6}	2.50
10^8	1.2×10^{-11}	10^{-20}	10^{-7}	2.54
10^{10}	1.2×10^{-12}	10^{-21}	10^{-10}	2.70

v_0 ranges from 10^2 m/s (for m = 10 amu), 10 m/s (for $m = 10^3$ amu), 2 m/s (for $m = 10^6$ amu) to 10^{-4} m/s (for $m = 10^{10}$ amu), 10^{-8} m/s (for $m = 10^{20}$ amu).

Quantum Violation of LGI for Larger Masses

The experimental constraint

As the mass is increased, it is found that in order to obtain significant QM violation of LGI, p_0 needs to be chosen such that both the classical amplitude of oscillation A_{Cl} and the required value of the initial peak v_0 for showing the QM violation of LGI become increasingly smaller. Thus, although, in principle, one can obtain the QM violation of LGI for any given m and ω by suitably choosing p_0, actual testability of this violation becomes gradually impracticable for sufficiently large mass, particularly beyond 10^{10} amu, as the requirement to controllably impart the required initial peak velocity becomes increasingly stringent ($< 10^{-4}$ m/s) and the required amplitude becomes smaller than 10^{-10} m.

Some salient points

- Comparing the calculational results with that using 3-term LGI, WLGI and NSIT, we find that for a given mass, the values of v_0 and A_{Cl} required to show significant violation of MR are *optimal* using 4-term LGI.
- If a pure coherent state is taken as the initial state, through interaction with environment, it becomes a mixture of coherent states, usually, the *thermal state*. In our example, even by taking the *thermal state* as the initial state, QM violation of LGI is found to *persist* corresponding to temperature ~ 0.1 K, whereas trapped nano-crystals to be used for the proposed experiment have been cooled to much lower temperature $\sim$ mK.
- QM violation of LGI in our example also persists for significant *unsharpness* of the spatial observable considered, and this robustness is of the same order as that obtained by using WLGI or NSIT.

The Proposed Setup

Experimental aspects of the proposed scheme

- The system considered is a nano-crystal of mass, say, 10^6 amu trapped by laser fields that generate a harmonic well of $\omega \sim 10^6$ Hz [Y. Bateman et al. *Nature Communications* 5, 4788 (2014)].
- Damping and decoherence effects are *negligible* for such a system in the experimental time-scale of $1/\omega (\sim 10^{-6} s)$ where the typical decoherence time is *1 - 10* ms for optically levitated oscillating objects.
- The positions of the *optically levitated masses* can be observed with extremely *high spatial resolution* by means of photo-diodes using the *interferometric (phase sensitive) detection* of light scattered from the objects.

  This enables detection of *positions* with *sub-Angstrom resolutions*, while the *detecting time-window* to achieve such resolutions is much *smaller* than the oscillation time-scale $1/\omega$ so that the measurements can be regarded instantaneous.
- By criss-crossing, say, the $x > 0$ domain of the well with scattering light fields whose intensity is ensured to fall to zero *sharply* at $x = 0$, in case the nano-object *fails* to scatter light, its state will be projected to the eigenstate of its localization within the $x < 0$ domain of the well.

To conclude......

The proposed setup (implementation considered by H. Ulbricht et al.) seems to be promising to provide an effective means for probing the macrolimit of the quantum world and for testing the everyday notion of classical realism for a system having classical analogue like a harmonic oscillator.